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SOME ASPECTS REGARDING THE PROBLEMS GENERATED BY THE DERIVATIVE CAUSALITY IN BOND-GRAPH MODELS

BY

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Abstract. The bond-graph model of a system could contain energy storage elements in derivative causality. The presence of these elements is not desirable, and whenever possible, it is advisable to avoid them. When they cannot be avoided, some difficulties in conceiving the state-space model and in the numerical simulation based on the block diagram model could occur. In the present work such difficulties are analyzed and solutions for them are presented.

Keywords: bond graph; derivative causality; derivative block; numerical simulation.

1. Introduction

The mathematical modelling of system dynamics could be performed by using different methods. One of them is the bond-graph method (Borutzky, 2010; Ibănescu, 2017; Karnopp *et al.*, 2006; Păstrăvanu and Ibănescu, 2001). The bond-graph model is a diagram which describes how the power in a system, provided by sources, is processed by the component elements of the system and transmitted to the working elements. This model can be exploited in three ways.

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A first possibility consists in conceiving a mathematical model called state-space model. Such a model contains only differential equations of first order and has as unknowns the energy variables corresponding to energy storage elements which are in integral causality, but there are no energy storage elements in derivative causality. When the bond-graph model contains energy storage elements in derivative causality as well, the mathematical model will consist of differential and algebraic equations. The number of differential equations equals the number of energy storage elements in integral causality, while the number of algebraic equations equals the number of energy storage elements in derivative causality. The differential-algebraic system could be transformed into a system of differential equations by using algebraic procedures which involve a great calculus volume (Ibănescu and Păstrăvanu, 2002).

The second possibility of exploiting the bond-graph model consists in obtaining, based on a well-founded procedure, a block diagram model which can be implemented into a proper software, like MATLAB-SIMULINK. If the bond-graph model contains energy storage elements in derivative causality, then the block diagram model will contain derivative blocks. These blocks frequently lead to numerical simulation blocking. In some circumstances, the problem could be resolved by replacing the derivative blocks with blocks of transfer function type, having the shape $\frac{s}{as+1}$, where a is a constant smaller than unity, whose value depends on the frequency of system oscillations (Ibănescu, 2011; Păstrăvanu and Ibănescu, 2001). This solution has no results when the number of derivative blocks is more than two. Practically, in this case, the numerical simulation based on the block diagram should be abandoned.

The third possibility of exploiting the bond-graph model is to directly plot it into a dedicated software, as 20 sim. In this case, the numerical simulation is a successful one, no matter the number of energy storage elements in derivative causality.

2. The Bond-Graph Model of a System with Nine Energy Store Elements in Derivative Causality

The mechanical system pictured in Fig.1 consists of a bar $2l$ in length, whose mass is M_1 and it has attached at one end a mass m and a spring of stiffness k_e and at the other end a disk of radius R and mass M_2, which rolls on a horizontal surface. A thread is attached to the centre of the disk and passes over two identical disks of radius R and mass M_3. At the ends of the thread hang two weights of mass M_4. This system is analysed in (Ibănescu and Ibănescu, 2017), where the bond-graph model (see Fig.2) and the mathematical model consisting in a system of differential-algebraic equations are also presented.

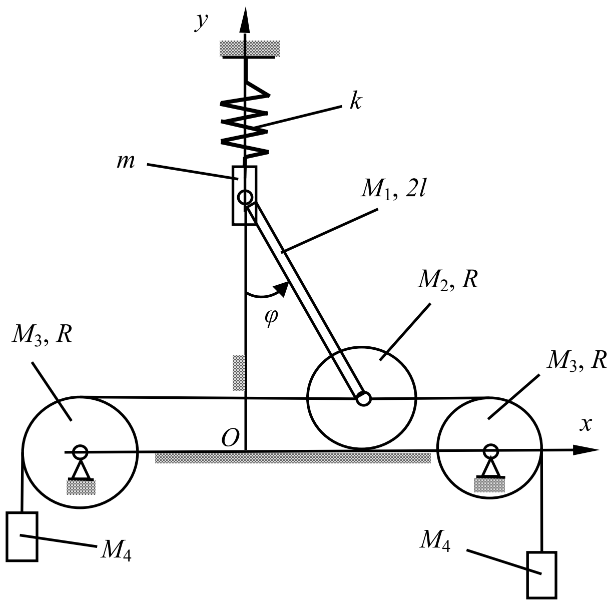

Fig. 1 – The mechanical system presented in (Ibănescu and Ibănescu, 2017).

This model contains two differential equations because there are two energy storage elements in integral causality (I_{10} and C_7) and nine energy storage elements in derivative causality (I_1, I_9, I_{13}, I_{16}, I_{19}, I_{22}, I_{26}, I_{29} and I_{33}). Some algebraic procedures are applied and finally a system of two differential equations is obtained, which is then numerically integrated.

3. The Simplified Bond-Graph Model

The bond-graph model presented in Fig. 2 could be significantly simplified. The two right branches could be substituted by a single one. In this manner, the bond-graph model shown in Fig. 3 is obtained. This model no longer contains nine energy storage elements in derivative causality, but only seven. This fact leads to the corresponding decreasing of the number of algebraic equations in the differential-algebraic system.

This change was possible because the two disks of mass M_3 and the two weights of mass M_4 have the same dynamic behaviour and could be considered as single bodies having double moment of inertia and double mass, respectively. The two effort sources which are equal to M_4g are balanced. The new simplified model leads to the same system of differential equations but it is much easier obtained.

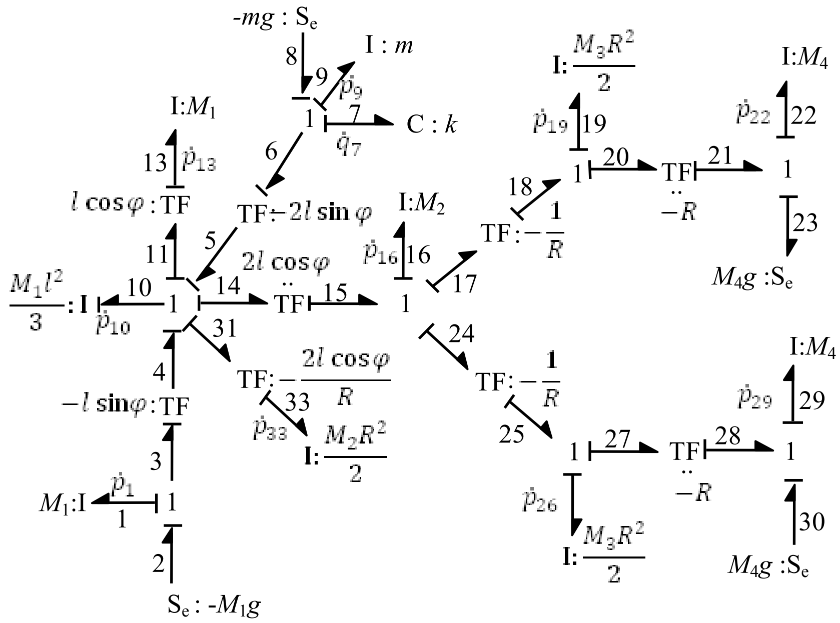

Fig. 2 − The initial bond graph model of the mechanical system.

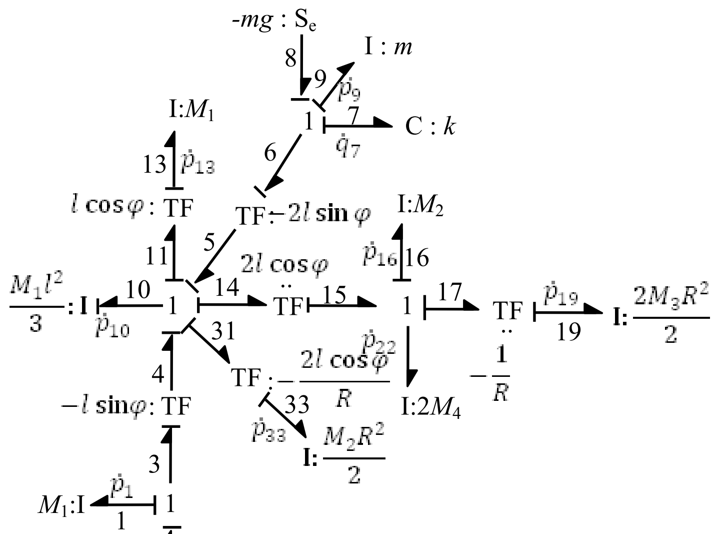

Fig. 3 − The simplified bond graph.

4. The Block Diagram Model

The block diagram model shown in Fig. 4 is based on the bond-graph model (Fig.3).

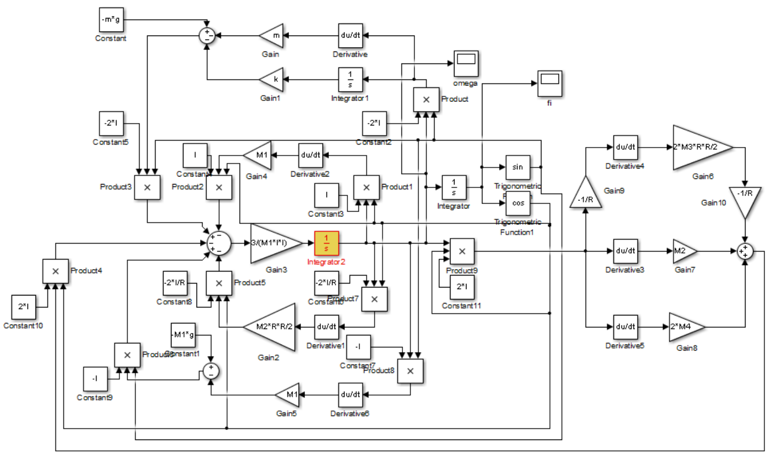

Fig. 4 − The block diagram model containing derivative blocks.

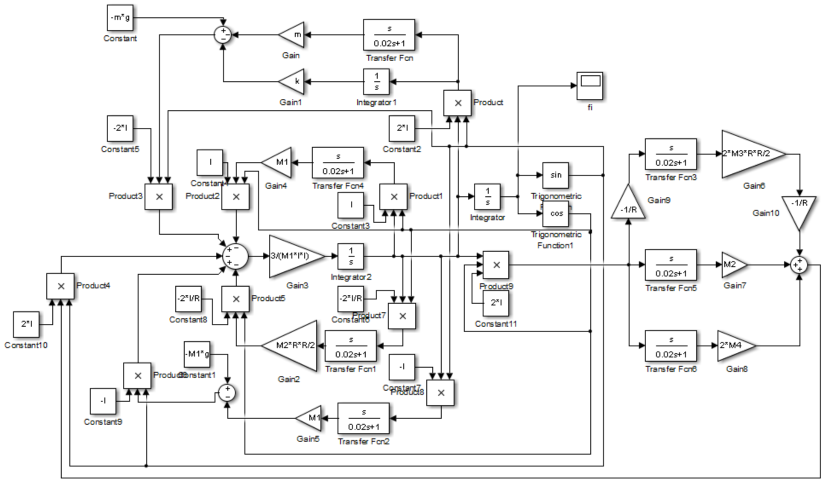

Fig. 5 − The block diagram model with transfer function blocks.

This model has seven derivatives blocks, which could be useful for the numerical simulation. By using the same numerical values as in work (Ibănescu and Ibănescu, 2017), that is: l = 0.5 m, m = 4 kg, M_1 = 8 kg, M_2 = 6 kg, M_3 = 6 kg, M_4 = 10 kg, g = 9.81 m/s^2, ϕ_0 = 0.6 *rad*., it is found that despite of reducing the number of derivative blocks, the numerical simulation fails and MATLAB error message is "*derivative input 1 of 'diagblocsimpla/Integrator2' at time* 4.2799999999999976E-5 *is Inf or NaN. Stopping simulation*".

The attempts of resolving this problem by modifying the parameters of the integrator block are doomed to failure.

Another solution is to replace the derivative blocks with blocks of transfer function type, having the shape $\frac{s}{0.02s+1}$ (Păstrăvanu and Ibănescu, 2001). The block diagram shown in Fig.5 results. The numerical simulation leads to the graph presented in Fig.6.

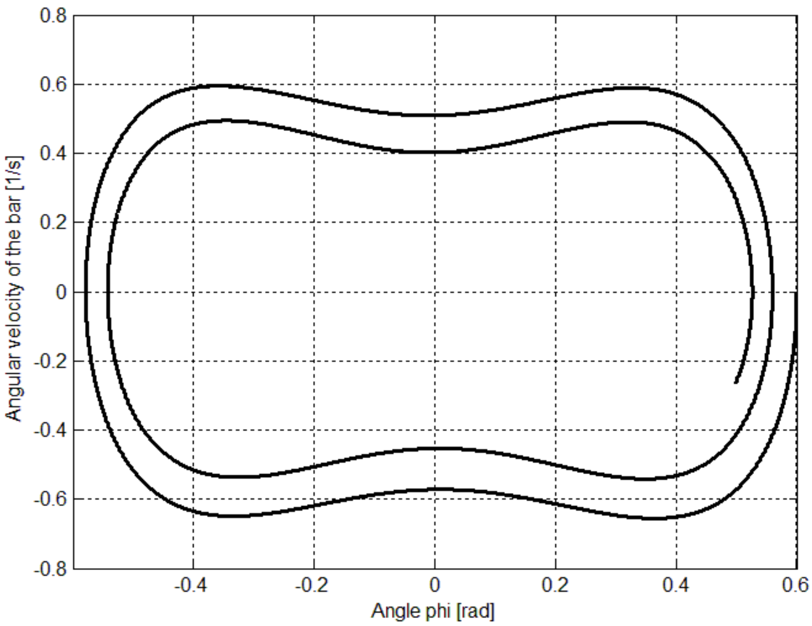

Fig. 6 – The simulation by using transfer functions.

This graph shows that the motion has a velocity loss, like for a non-conservative system. This is the effect of the error introduced by the derivative blocks. The shape of the graph is like that one which results by integrating the system of differential equations. The numerical simulation results are not accurate enough for practical use.

5. The Bond-Graph Model Plotted in a Proper Soft

The bond-graph model pictured in Fig. 3 can be transferred into a proper soft. The bond-graph model transferred in the soft 20 sim is pictured in Fig.7.

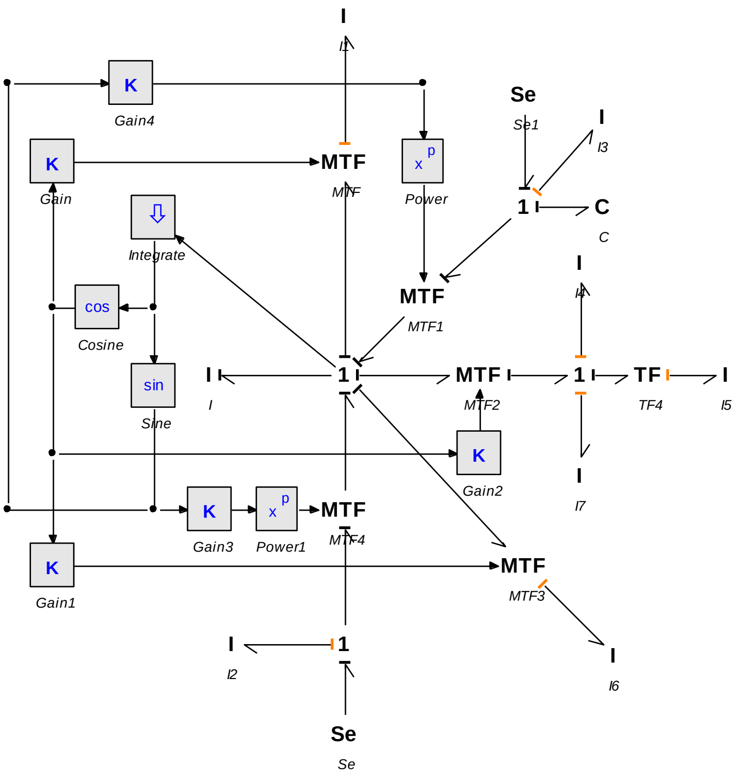

Fig. 7 – The simplified bond graph model represented in 20 sim.

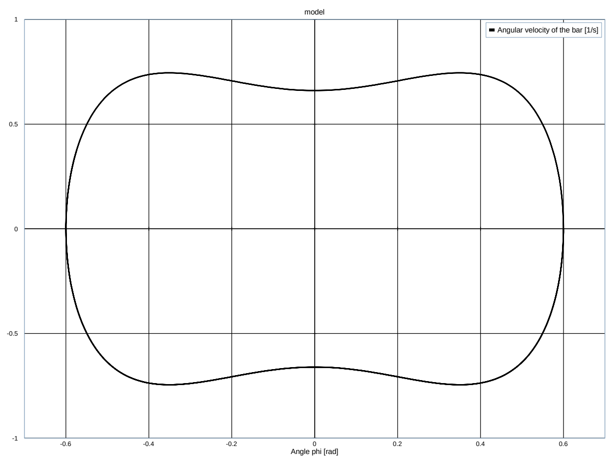

Fig. 8 – The simulation obtained in 20 sim.

This is the simplest possibility of exploiting the bond-graph model and it is not influenced by the presence of energy storage elements in derivative causality.

6. Conclusions

1. The bond graph model which contains energy storage elements in derivative causality leads to different difficulties in obtaining the system of differential-algebraic equations.

2. Any bond-graph model could be analysed in order to simplify it. The model simplifying is based not only on the classical rules of conceiving the bond-graph model but also on the analysis of system functioning.

3. The existence of energy storage elements leads to derivative blocks in the block diagram model. In case of a great number of such blocks, the numerical simulation results are not accurate enough, even when the derivative blocks have been replaced by transfer functions of form $\frac{s}{as+1}$. In this case a = 0.02 because the oscillations frequency is low. The simulation shows only the general shape of the diagram but an accurate result is not obtained.

4. When the model contains energy storage elements in derivative causality, the use of a proper software represents an efficient solution. This example points out the advantages of the first two possibilities, which are:

a) There is no need to write and solve differential equations;
b) The orientation of the bonds is automatically assigned;
c) The causal stroke is automatically assigned;
d) It is needed to write only the parameters of the elements and the initial conditions;
e) The numerical simulation runs no matter if there are accumulative blocks in derivative causality.

REFERENCES

Borutzky W., *Bond Graph Methodology*. *Development and Analysis of Multidisciplinary Dynamic System Models*, Springer, 2010.

Ibănescu R., *Bond Graphs in System Modelling* in *Graph Based Modelling in Engineering*, 3-23, Springer, 2017.

Ibănescu R., Ibănescu M., *Aspects of Derivative Causality in Bond-Graph Models*, MATEC Web of Conferences **112**, 06007 DOI: 10.1051/matecconf/201711206007 *IManE&E 2017* (2017).

Ibănescu R., *The Bond-Graph Model Containing Derivative Causality*, Proceedings of 15th International Conference Modern Technologies, Quality and Innovation, Chișinău, Republic of Moldova, 493-496 (2011).

Ibănescu R., Păstrăvanu O., *Numerical Problems in Block-Diagram Simulation of Bond-Graph Models with Derivative Causality–*A MATLAB-SIMULINK-*Based Case Study*, Bul. Inst. Polit. Iaşi, s: Mathematics. Theoret. Mech. Phys., **XLVIII (LII)**, *1–2*, 75-86 (2002).

Karnopp D.C., Margolis D.L., Rosenberg R.-C., *System Dynamics. Modeling and Simulation of Mechatronic Systems*, 4th Edition, Willey, 2006.

Păstrăvanu O., Ibănescu R., *Bond-Graph Language in Modeling and Simulation of Physical-Technical Systems*, "Gheorghe Asachi" Publishing House, Iași , 2001.

UNELE ASPECTE PRIVIND PROBLEMELE GENERATE DE CAUZALITATEA DERIVATIVĂ ÎN MODELELE BOND GRAPH

(Rezumat)

Modelul bond graph al unui sistem poate conține elemente acumulatoare de energie aflate în cauzalitate derivativă. Prezența acestor elemente nu este de dorit și, ori de câte ori este posibil, prezența lor trebuie evitată. Când prezența acestor elemente nu poate fi evitată, vor apărea dificultăți în rezolvarea sistemului de ecuații care modelează dinamica sistemului dar, mai ales, în simularea numerică bazată pe modelul bond graph. Lucrarea prezintă și analizează aceste dificultăți și prezintă soluții pentru rezolvarea lor.